When you heat a rubber band it contracts. If you only know about point particles and ideal gases, this behavior is perplexing. But a simple classical statistical mechanics model of a chain provides some insight.

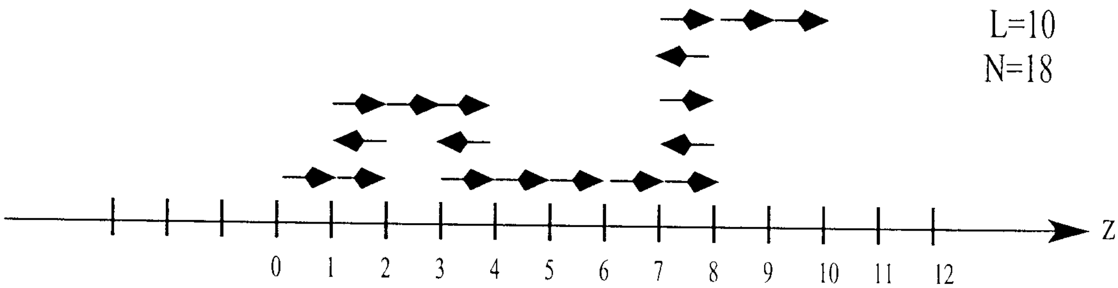

FIG. 1. Random Walk Model of a Rubber Band.

Model the rubber band as a single chain of N links, with each link having fixed length a and mass m which resides at the link's tip. Treat the chain as a one dimensional random walk along the z axis. Let one end be pinned at the origin and let each link proceed to the left or the right with equal probability. Suppose the other end of the chain is located a distance La from the origin, where L could range freely from $-N$ to $+N$. There are no interactions between links and the links can even overlap at no cost, as shown in the figure.

(a) Derive an exact formula for the number of configurations of the chain of given N and L. Call the result $\Omega(N, L)$. Set the length a of each link to unity for convenience.

Knowing $\Omega(N, L)$, we can calculate the entropy, $S = k \ln \Omega(N, L)$, where k is Boltzmann's constant, and do some statistical mechanics. Accept the fact that for large N and L, with $N \gg L$, $\Omega(N, L) \approx C(N) \exp(-L^2/2N)$. The constant $C(N)$ will not be important here. Use this approximate form for $\Omega(N, L)$ in the rest of this model-building problem.

Now put gravity into the problem by supposing there is a force mg acting on each link and pointing in the $+z$ direction.

(b) Write down the expression for the Free Energy of the chain, show that it satisfies Hook's Law and identify the temperature dependence of Hook's coefficient.

(c) Sketch the Free Energy vs. L at fixed T and explain how the minimum represents a competition between order and disorder.

(d) Find the minimum of the Free Energy of the chain as a function of L. Show that the result predicts that rubber bands contract when heated.