Consider a ferromagnet of volume V in which the spin of each atom is aligned in the z direction at temperature $T = 0$. For $T > 0$ spins fluctuate about the z-axis and acquire small transverse components (S_x, S_y) which obey the continuum equations of motion,

$$\frac{\partial S_x}{\partial t} = J\nabla^2 S_y \qquad\qquad \frac{\partial S_y}{\partial t} = -J\nabla^2 S_x$$

where J is a constant determined by the interactions between spins.

(a) Show that these spin fluctuations obey the dispersion relation

$$\omega = J\left(k_x^2 + k_y^2 + k_z^2\right)$$

where $\vec{k} = (k_x, k_y, k_y)$ is the wavevector.

(b) By quantizing the spin fluctuations one obtains magnons. Magnons are noninteracting bosons that are not conserved. Show that the density of states in energy for magnons in three dimensions takes the form,

$$D(E)\, dE = A V E^{3/2}\, dE$$

Find the constant A in terms of J and Planck's constant.

In the following parts you may express your answers in terms of the dimensionless integrals

$$I_1 = \int_0^\infty \frac{x^{1/2}}{e^x - 1} dx \qquad\qquad I_2 = \int_0^\infty \frac{x^{3/2}}{e^x - 1} dx$$

(c) Find the mean number of magnons in the solid at temperature T.

(d) Find the mean energy density of magnons at temperature T.

(e) Find the magnon pressure at temperature T.