A narrow wire of length L can be considered to be a one- dimensional potential-free region between two walls of infinite potential at $x = 0$ and $x = L$. You may ignore the (electrostatic) interactions between the multiple electrons that fill the wire. Let k_B denote Boltzmann's constant.

(a) Show that electrons of mass m which occupy the wire have allowed solutions of the Schrödinger equation with energies given by:

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2,$$

where n is a positive integer.

(b) The electrons in the wire are allowed to come to equilibrium with a reservoir of electrons of chemical potential μ at temperature T. Write down an inequality involving L, μ, m, k_B and T that determines when the statistical physics of the system cannot be considered classical: this is the situation in which all states would be almost completely occupied or completely empty. Such a system is sometimes called a "quantum wire".

(c) Now consider first the limit in which $\mu \gg E_1$ such that many electrons fill the wire. Draw a sketch graph of the occupation of the states as a function of their energy. Determine with the aid of your sketch (or otherwise) whether the number of electrons in the wire increases or decreases with T. It is not necessary to use any equations.

(d) Under these same assumptions, show that the number of electrons in the wire as a function of μ and T is given by:

$$N(\mu, T) = A \int_{-\alpha}^{\infty} \frac{dt}{(1 + e^t)(\alpha + t)^{1/2}}$$

where

$$\alpha = \frac{\mu}{k_B T}$$

and A is a constant for which you should give an expression.