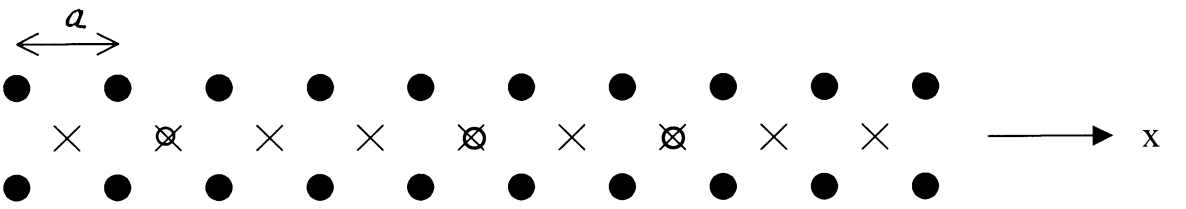
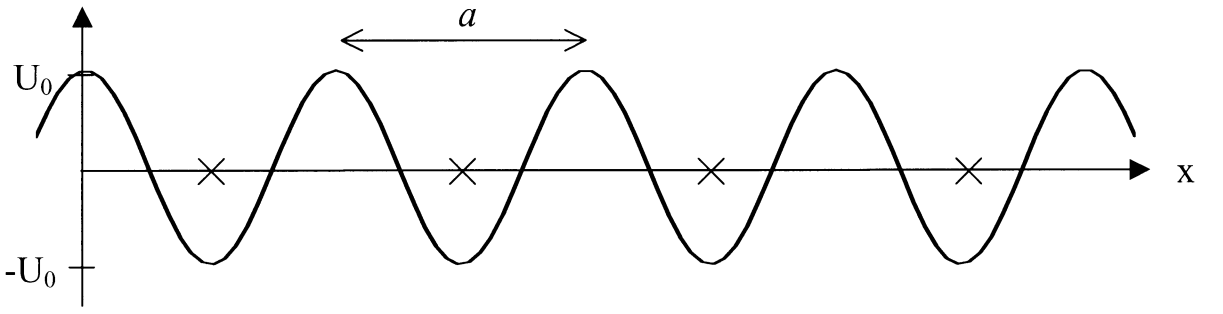
Consider a simple classical model for ionic conduction in a one-dimensional crystal.



A density n (number per unit length) of ions (O) with mass m and charge q occupy the interstitial sites (×) between atoms (●) in a one-dimensional chain. Assume the density is low ($n \ll 1/a$) and that at most one ion can occupy each interstitial site. The distance between atoms is a.

The interstitial ions can move by hopping between adjacent interstitial sites in the periodic potential $U(x) = U_0 \cos(2\pi x/a)$ shown below.



(a) Find the small oscillation frequency of an ion in one of the interstitial sites.

(b) At temperature T ($k_B T \ll U_0$), find the thermally-activated hopping rate (number of hops per unit time) between adjacent minima.

An electric field E ($qEa \ll U_0$) is applied in the x-direction.

(c) Find the ratio of the hopping rate in the positive x-direction to that in the negative x-direction.

(d) Find the electric current I in the x-direction for electric field E and temperature T.