SMFall02A

This problem illustrates the basic features of the statistical mechanics of thermal expansion and related phenomena.

Consider a one-dimensional chain of length L, made of N one-dimensional harmonic oscillators. We assume that the energy of the chain is given by

$$E = \frac{\alpha}{2}\left(L - L_o\right)^2 + \sum_{i=1}^{N}\left(N_i + \frac{1}{2}\right)\hbar\omega$$

where the angular frequency, ω, is a function of L, N_i is the occupation number for the i^{th} 1-dimensional harmonic oscillator and L_o and α are both constants. Give your answers in terms of ω and $d\omega/dL$.

a) Assuming the chain is of finite length, determine the equilibrium length of the chain at T=0

b) Find the Helmholtz free energy, F, of the chain.

c) Suppose now that $\omega(L) = \alpha/L$. Calculate the coefficient of thermal expansion.