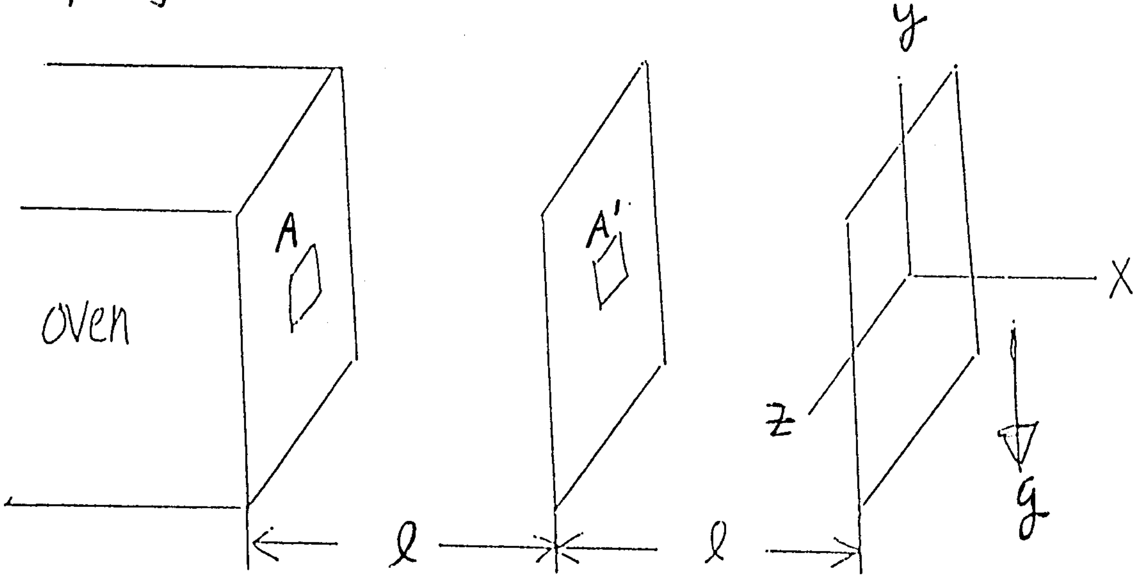
SM Spring 02B



An oven contains n_0 molecules/volume of a classical gas at a temperature T. The molecules have mass m. As shown in the figure, there is a small square opening of area A in the oven, through which molecules are emitted into the evacuated region outside the oven.

(a) What is the number of molecules/sec emitted from the oven?

A distance ℓ away from the oven is a wall with a small square opening of area A'.

A distance ℓ beyond the first wall is a second wall. Molecules are emitted from the oven, pass through the opening A' in the first wall, and strike the second wall at y, where $y = 0$ corresponds to the center of the openings A and A'. Gravity acts in the $-y$ direction, and the acceleration due to gravity is g. The geometry is such that ℓ is very large compared to y, and the linear dimensions of the two holes A and A' are both very small compared to either y or ℓ.

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NOTE: In the following, v_x, v_y, v_z mean initial x, y, z velocities, just after the molecule is emitted from the oven.

(b) A molecule is emitted from the oven and passes through the center of the opening A'. By considering the dynamics of its motion between the oven and opening A', derive a formula for v_y in terms of v_x.

(c) The same molecule having passed through the center of A', strikes the wall at y. Use this information and your result in (b) to derive a formula for v_x^2 which involves only g, ℓ, and y.

The number of molecules/sec striking the second wall between y and $y + dy$ can be written as

$$n_0 A\, A' \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} \frac{g^2 \ell^2}{2|y|^{\beta}} \exp\left(-\frac{mgh(y)}{2kT} \right) dy$$

(d) By using dimensional analysis, determine the exponent β.

(e) Determine the function $h(y)$ in terms of g, ℓ, and y. Hint: Make use of the Boltzmann factor for the molecule just after it is emitted from the oven.