SM FALL 01 A

A crystal contains N nuclei that do not interact with each other. Each nucleus can be considered to have angular momentum $L = 1$. In the crystal the states for each nucleus can be labeled with $m = -1, 0, +1$ quantized along a crystal axis, but there are perturbations due to the internal structure of the crystal that split the levels, so that the energies of the states for each nucleus in the crystal are $E = 0$ for $m = 0$ and the same positive value $E = \varepsilon > 0$ for $m = +1$ and $m = -1$.

a) Without doing a detailed calculation, state how the entropy should scale with N for large N.

b) Derive an explicit expression for the entropy S as a function of temperature T. Give your answer in terms of ε and do not leave any summations or integrals in your expressions.

c) Derive the lowest order approximation for the heat capacity valid at low temperature, $k_B T << \varepsilon$ and at high temperature $k_B T >> \varepsilon$. (You do not need to give an expression valid at all temperatures.)

d) This question concerns the qualitative nature of the perturbing Hamiltonian in the crystal that leads to the energy levels given in the statement of the problem. Could a uniform magnetic field present at each nucleus give rise to the given energy levels? Briefly explain your reasoning.