SM 7all 00 B

Consider a system of N spin *one* degrees of freedom, at temperature T. The magnetic moment of the i-th spin ($i = 1, \ldots, N$) $\vec{m}_i$ and its spin $\vec{S}_i$ are related by $\vec{m}_i = |m|\vec{S}_i$. The eigenstates of the z-component of the spin are $|S_i\rangle = |\uparrow\rangle, |0\rangle, |\downarrow\rangle$, with eigenvalue $S_i = +1, 0, -1$ respectively (the factors of $\hbar$ are absorbed in the coupling constant J used below). The configurations of the system can be labelled by the z-component of each spin, and in that basis can be written in the form $|S_1, \ldots, S_N\rangle$. The spins of the system are interacting only with an external magnetic field $\vec{B}$ pointing along the z axis. The total energy E is

$$E = -\sum_{i=1}^{N} JBS_i$$

We will also define the magnetization per spin, $M = \frac{1}{N}\sum_{i=1}^{N} S_i$.

A) Write down expressions for the *probability* of each one of the following configurations at temperature T:

i) all spins are up, *i. e.* $|\uparrow, \ldots, \uparrow\rangle$.

ii) $\frac{1}{3}$ of the spins are up, $\frac{1}{3}$ of the spins are zero, and $\frac{1}{3}$ of the spins are down, (assume that N is a multiple of 3).

B) Calculate the partition function Z for this system.

C) Find explicit expressions as a function of temperature for the following physical quantities:

i) the free energy F per spin.

ii) the internal energy $\langle E\rangle$ per spin.

iii) the specific heat c.

iv) the average magnetization per spin $\langle M\rangle$.

D) Calculate the leading order asymptotic behaviors of the magnetization per spin, $\langle M\rangle$, and the specific heat c at low and high temperatures. Give explicit inequalities that the temperature T has to satisfy in each regime, expressed in terms of the physical parameters of the total energy E.