SM Spring 00B

Consider a potential well with a non-degenerate ground state level of energy zero, and 2 excited levels with the same energy ε. These are the only levels of the well.

(a) Place 3 identical spin-1/2 Fermi-Dirac particles, all with spin up, in the well at temperature T.

1) Let g_i be the number of ways to distribute $i = (0,1,2,3)$ particles in the excited states. Find g_0, g_1, g_2, and g_3.

2) Write out the partition function of the system in terms of the g_1 found above, ε and k_BT, where k_B is Boltzmann's constant.

3) Determine the probability of finding N_0 (=0,1,2,3) particles in the ground state of the well.

4) Determine the mean number of particles $\langle N_0 \rangle$ in the ground state level, and the relative fluctuations $(\langle N_0^2 \rangle - \langle N_0 \rangle^2)/\langle N_0 \rangle$ of the occupation of the ground state level.

(b) Now instead place 3 identical spinless Bose-Einstein particles into the same well at temperature T.

1) Find g_0, g_1, g_2, and g_3, where as above g_i is the number of ways to distribute i particles in the excited states.

2) Write out the partition function of the system in terms of the g_i, ε and k_BT.

3) Determine the probability of finding N_0 (=0,1,2,3) particles in the ground state of the well.

[continued on next page]

4) Calculate explicitly at high temperature to first order in $x = \varepsilon/k_B T$:

i) the partition function;

ii) the mean number of particles $\langle N_0 \rangle$ in the ground state level; and

iii) the relative fluctuations $(\langle N_0^2 \rangle - \langle N_0 \rangle^2)/\langle N_0 \rangle$ of the occupation of the ground state level. Why, briefly, does $\langle N_0 \rangle$ approach unity at high temperatures?