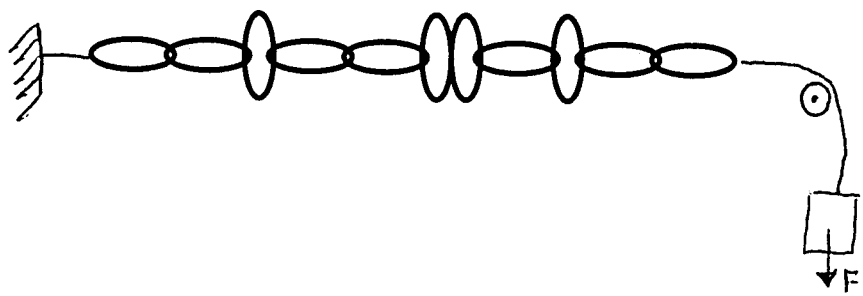
SMSpring00A

A simple model of a rubber band is a one-dimensional (horizontal) chain consisting of N (N>>1) linked segments, as shown in the diagram. Each segment has just two possible states: horizontal with length a, or vertical, contributing nothing to the length. The segments are linked such that they cannot come apart. The chain is in thermal contact with a reservoir at temperature T.

(a) If there is no energy difference between the two states, what is the average length of the chain?

(b) The chain is now fixed at one end and a weight hung from the other end, supplying a force F as shown below. Determine the average length of the chain at any temperature T. Find the length in the limits $T \to 0$ and $T \to \infty$.



(c) In which temperature limit is the extension proportional to F (Hooke's Law)? Calculate the constant of proportionality.

(d) As the temperature is raised, does it get harder or easier to extend the rubber band? If you warm up the rubber band, while supplying a constant force, will it contract or expand? Give a qualitative explanation of the behavior in terms of the dependence on length of the Helmoltz free energy.