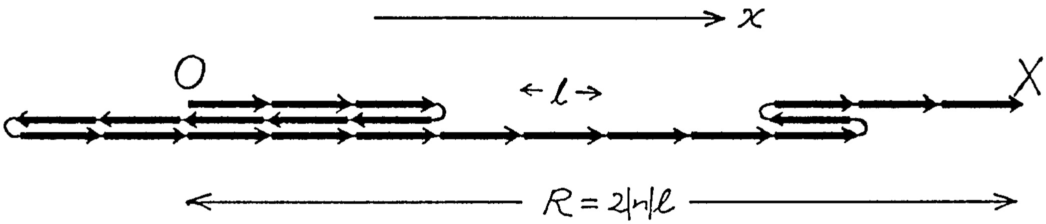
SM Fall 99 B

Let us consider a polymer consisting of N (a positive even integer) monomers connected end-to-end and lying along the x-axis as shown in the figure. The monomers may be regarded as small arrows of lengths ℓ without thickness and lie along the x-axis with no preferred direction. One end of the polymer is fixed at the origin O.



(a) Find the number N_+ of monomers pointing in the positive direction along the x-axis, and the number N_- of monomers pointing in the negative direction along the x-axis, when the free end x-coordinate is $X = 2n\ell$, where $-N/2 \leq n \leq \mathrm{N}/2$.

(b) Write down the total number $\Omega_N(X)$ of allowed conformations when the free end x-coordinate is $X = 2n\ell$, as in (a).

(c) Let $S_N(X)$ be the conformational entropy of our polymer whose end x-coordinate is X. Compute the entropy difference $S_N(X) - S_N(0)$ to order X^2, assuming that $|n| \ll N$. You may use

$$M! \simeq \sqrt{2\pi} e^{-M} M^M.$$

[continued on next page]

(d) Find the required average force in pN (picoNewton $= 10^{-12}$N) to make the end-to-end distance of the polymer be $R = 200$Å. Assume that $N = 4 \times 10^4$, the monomer size ℓ is 1Å($= 1\times10^{-10}$m), and the ambient temperature is 300K. Assume that the only contribution to the forces comes from the conformational entropy of the polymer for which you may use the functional form obtained in (c). Do not hesitate to use thermodynamic considerations, if microscopic calculations can be avoided.