SM7a1198B

Consider the rotational motion of a dilute gas of N diatomic molecules, each with moment of inertia I. In this problem you are asked to consider three cases: a quantum system of diatomic molecules in which the two atoms are distinguishable (parts A-C); a quantum system of diatomic molecules in which the two atoms may be considered as indistinguishable spinless bosons (part D), and a classical system of rotors (part E). In this problem we will consider rotational motion only. You may use the fact that a quantum rotor has energy levels $E_J = \frac{\hbar^2}{2I} J(J+1)$.

In parts A-C, use quantum statistical mechanics for the case where the two atoms in each molecule are distinguishable

A) Give an expression for the partition function Z as a function of temperature T. You do not need to evaluate the expressions in a closed form. Be sure to properly include all degeneracy factors in your expression.

B) When the temperature T is low but non-zero, derive an asymptotic expression for the rotational specific heat per molecule C as a function of temperature.

C) In the limit of high temperature, the expression for Z in part (A) can be approximated as an integral over a continuous variable. By evaluating the integral, give the limiting value of the specific heat per molecule C.

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In part (D) consider the case where the atoms in each molecule can be regarded as indistinguishable spinless Bosons.

D) Give the expression for the partition function Z in this case which replaces the one in part (A). Explain your reasoning. You do not need to evaluate the expression.

In part (E), consider the relation to classical statistical mechanics.

E) The expression for the specific heat using quantum statistical mechanics should approach the classical result in the high temperature limit. Show that your result of part (C) does approach the correct classical limit in which the specific heat is determined by the number of degrees of freedom for the rotational motion.