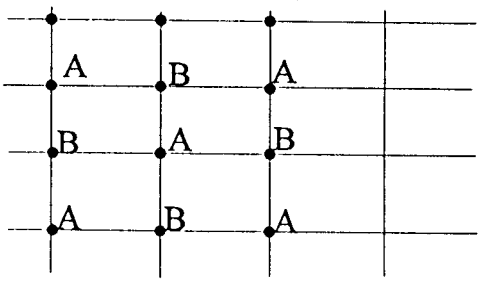
SM 7a1198A

N identical classical particles occupy a square lattice with 2N sites, with at most one particle per site. Alternate sites are labeled A and B, as sketched below. Denote by c the fraction of particles on the A sites.



A) For fixed c, and assuming that all configurations at fixed c are equally likely, (a mean field approximation) calculate the entropy S(c) of the system for large values of N. Evaluate the entropy when c = ½.

B) When two objects are on neighboring A and B sites, there is a repulsive interaction energy E_0. For fixed c, and assuming that all configurations at fixed c are equally likely, show that the average total energy of the system is

$$E(c) = 4NE_0\, c(1\text{-}c).$$

In thermal equilibrium at temperature T, c is determined by minimizing the free energy F(c) = E(c)-TS(c). This system exhibits a second order phase transition at a temperature T_c.

C) Describe the state of the system at very high temperatures. What is the observed value of c?

D) Describe the state of the system at very low temperatures. What are the possible values of c?

E) Determine T_c.

HINT: For large values of N we can approximate lnN! by NlnN-N.

For small x, $\ln(1 + x) \approx x - x^2/2 + x^3/3 + \ldots$.