SM Spring 98B

A gas of N indistinguishable classical non-interacting atoms of mass m is held in a neutron atom trap by a potential of the form $V(r) = ar$, where $r = \sqrt{x^2+y^2+z^2}$. The gas is in thermal equilibrium at a temperature T.

In this problem, we will work at the semi-classical level: the gas is considered classical, but we will take into account the indistinguishability of the atoms (a quantum effect) and the fact that the volume of classical phase space per quantum state is h^3 in three dimensions, where h is Planck's constant.

(a) Find the partition function Z_1 for a single atom trapped in this potential. Express your answer in the form $Z_1(T,a) = AT^{\alpha}a^{-\eta}$. Find the exponents of α and η, and the constant A in terms of m, k_B, and Planck's constant h.

(b) Find the entropy of the gas in terms of N, k_B, and $Z_1(T,a)$. Do not leave any derivatives in your answer.

(c) The gas can be cooled if the potential is lowered reversibly (by decreasing a) while no heat is allowed to be exchanged with the surroundings (i.e. $dQ = 0$). Under these conditions, find T as a function of a and the initial values of T_0 and a_0.