SMSpring 98A

In this problem, we will be exploring the Bose-Einstein transition temperature, T_{BE}, of a gas of N non-interacting spinless Bose particles. Below T_{BE}, there is macroscopic occupation of the ground state. Here, each of the particles has mass m and all are enclosed in a three-dimensional volume V at temperature T.

(a) Find an expression for the density of available single-particle states $D(\varepsilon)$ as a function of the single-particle energy ε. Sketch $D(\varepsilon)$, being careful to label the axes and the origin.

(b) What is the allowed range of μ for a non-interacting Bose-Einstein gas?

(c) Write down an expression for the mean occupation number of a single particle state, $\langle n \rangle$, as a function of ε, T and $\mu(T)$, where μ is the chemical potential and $T_{BE} < T < \infty$. Sketch $\langle n \rangle$ at temperature T as a function of ε. Mark the location of μ.

(d) Write down an integral expression which implicitly determines $\mu(T)$. As the temperature, T, is lowered, how does $\mu(T)$ change? It will help to refer to your sketch.

(e) Using your answer to (d), find T_{BE}.

You may find this useful: $\int_0^\infty dx \frac{x^{1/2}}{e^x - 1} = 1.306\,\pi^{1/2}$