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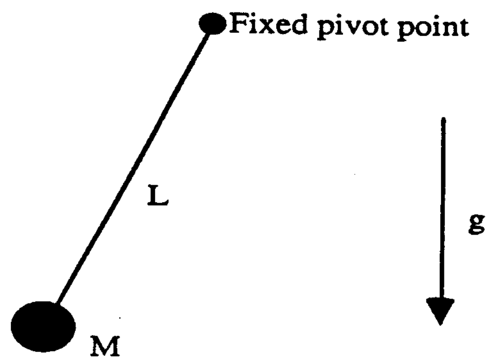

Consider a spherical pendulum made of a rigid massless rod, length L, supported at one end and free to pivot in all directions (including over the top) with mass M at the other end, in a gravitational field g. The pendulum is in thermal equilibrium with the surroundings at temperature T. (For the purposes of this problem, ignore relativistic effects.)

(a) State a condition on g, L, M, T, k_B and $\hbar$ (not worrying about numerical factors) for the statistical mechanics to be treatable in the classical (non-quantum) approximation.

(b) State a condition on g, L, M, T, k_B and $\hbar$ (not worrying about numerical factors) for the statistical mechanics to be treatable in the harmonic-oscillator approximation.

(c) Assuming that both the above conditions are met, give the expected kinetic energy $\langle K \rangle$ and the expected potential energy $\langle U \rangle$. Let $U=0$ when the pendulum hangs straight down.

(d) Give the leading asymptotic terms for $\langle K \rangle$ and $\langle U \rangle$ in the limit of large T.

(e) Only assuming that the classical approximation holds, find $\langle K \rangle$ and $\langle U \rangle$ for arbitrary T.