SMSpring97A

A sample consists of N independent electric dipoles. Each dipole has two possible quantum states with energies $\pm\mu E$, where E is the magnitude of an externally applied electric field. The lower energy state has dipole moment μ and the higher energy state has dipole moment $-\mu$.

(a) Find the total electric dipole moment of the sample in an electric field E and at temperature T.

(b) Show that the entropy of the sample is given by,

$$S = Nk_B\left\{-\frac{\mu E}{k_B T}\tanh\left(\frac{\mu E}{k_B T}\right)+\ln\left[2\cosh\left(\frac{\mu E}{k_B T}\right)\right]\right\}$$

where k_B is Boltzmann's constant. You may use the fact that $S=-\left(\frac{\partial F}{\partial T}\right)_V$ where F is the Helmholtz free energy.

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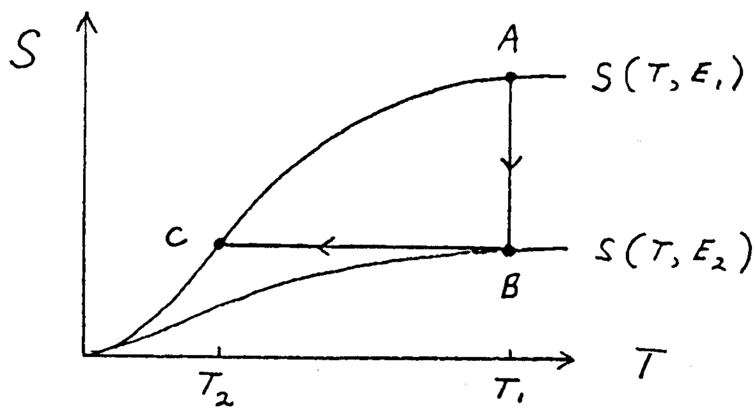
(c) Without using the formula in b, explain physically what the entropy should be in the limits $E \to 0$ and $E \to \infty$.

Entropy versus temperature curves for two values of electric field are shown below. Imagine that the sample is initially at state A, with temperature T_1 and field E_1.



(d) How much heat must be extracted from the sample to move it from state A to state B, maintaining its temperature at T_1 while the field is raised from E_1 to E_2?

(e) Once the sample is in state B, it is thermally isolated and the field is slowly reduced from E_2 to E_1, bringing the system from state B to state C. What is the temperature of the sample once it reaches state C?