SM7a/1963

A nucleon, which carries one unit of "baryon number," is a bound state of three quarks, each carrying one third of a unit of baryon number. If one compresses a gas of nucleons to sufficiently high density, the nucleons will unbind and turn into a gas of quarks. Both quarks and nucleons are spin-1/2 particles obeying Fermi statistics.

The aim of this problem is to construct a simple model of the phase transition at zero temperature from a nucleon gas to a quark gas. In this problem neglect all internal quantum numbers of the nucleon (proton or neutron), and of the quark (color and flavor), other than spin. Also, assume that the nucleons are a non-interacting *non-relativistic* gas, and that the quarks are *fully relativistic particles* so that the kinetic energy of a quark of momentum p is cp, where c is the speed of light.

(a) Consider only zero temperature where the nucleon and quark gases are both degenerate.

(i) Give the density of the nucleon gas, n_n, in terms of its Fermi momentum, p_n, and give the mean energy, E_n, per nucleon in the gas in terms of n_n.

(ii) Give the density, n_q, of the quark gas in terms of the quark Fermi momentum, p_q, and give the mean kinetic energy, K_q, per quark in terms of n_q.

(iii) If a gas of nucleons unbinds at a given fixed density, n_n, the corresponding density n_q of the quark gas equals $3n_n$. How are the nucleon and quark Fermi momenta at this density related?

(b) Assume that the quarks prefer to be bound in nucleons because the unbound state of quarks has an extra energy per unit volume, B, a positive constant. The total energy per quark in the quark gas is thus $E_q = K_q + B/n_q$.

(i) Show that at low densities the nucleon gas has a lower energy per baryon than the quark gas, while at very high densities the quark gas has the lower energy per baryon;

(ii) sketch the energies per baryon in the two phases as a function of baryon density, clearly labelling the curves and the baryon density, n_T, at which the system undergoes a transition from the nucleon to quark phase;

(iii) write down the equation determining the transition density n_T between the two phases.