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Ruby is a crystal of Al_2O_3 doped with chromium Cr^{3+} ions. Due to interactions with Al_2O_3, the Cr^{3+} ion has a magnetic state of $^4F_{3/2}$ in spectroscopic notation $^{2S+1}L_J$. The Hamiltonian in zero field is given by:

$$H = D[S_z^2 - (1/3)\, S(S+1)],$$

where the constant $D > 0$, and $\vec{S}$ is the total spin operator of the Cr^{3+}. Consider a sample of ruby containing N chromium Cr^{3+} ions.

(a) Find the energy levels and their degeneracies.

(b) Calculate the probability of occupancy of the lower energy state at temperature T.

(c) Calculate and sketch the entropy of the Cr^{3+} ions as a function of temperature. Find the limiting values at low T and high T.

(d) Find and sketch the heat capacity from the Cr^{3+} ions as a function of temperature. Find the functional forms at low T and high T.