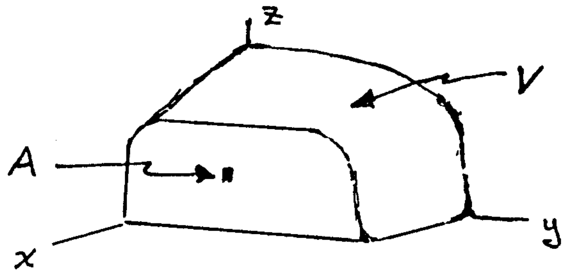
SMSpring96A

A perfect gas of N molecules is in equilibrium in a container of volume V, at a temperature T.

(a) What is the velocity probability distribution function, P(v), for this gas? Using the integral information at the end of this problem, obtain the proper normalization of P(v).

At time t=0, a very small hole of area A is punctured in a wall of the container that lies in the (y-z) plane (see the figure). The rate of leakage is slow enough for the gas within the container to remain in thermal equilibrium at all times.



Two situations will be considered: (1) when T is constant (an isothermal process), and (2) an adiabatic process (when there is no energy exchanged between the container and the gas).

(b) In the isothermal case, show that the equation for dN/dt, the rate of change of the number of gas molecules in the container per unit time, is of the form

$$dN/dt = C\,(N/V)\,A\,(k_BT/m)^{1/2}\,,$$

where C is a pure number. Do this by expressing dN/dt in terms of an integral involving P(v), the instantaneous probability velocity distribution function. Using the integral information below, determine the constant C.

(c) In this isothermal case, assume now that the container holds two perfect gases with molecular masses $m_1 > m_2$, and $N_1 = N_2 = N_0$ at t=0. Without performing any further calculations, explain what will happen to the ratio (N_1/N_2) as t increases.

(d) For the case (c), obtain the equation for the rate of change, $d(N_1/N_2)/dt$, and determine (N_1/N_2) as a function of time, using your previous results.

(e) If a monatomic perfect gas leaks out in an adiabatic process, will the temperature of the gas inside the container rise, fall, or remain constant in time? Explain your reasoning.

(f) Suppose the gas leaks out in an adiabatic process. Let U be the internal energy of the gas. Express the rate of change of the internal energy of the gas in the container in terms of integrals involving the instantaneous equilibrium probability velocity distribution function, P(v,t). Explain your reasoning. Obtain the dependence of dU/dt on N and T.