SM Spring 95A

Consider a system of many identical noninteracting spinless fermions occupying a large number M of distinct single-particle states. Of these single-particle states, $M/3$ have energy $-\varepsilon$, $M/3$ have energy zero and $M/3$ have energy ε.

(a) Calculate the partition function and the mean number of particles $\langle N \rangle$ in the grand canonical ensemble, in terms of the chemical potential μ and the temperature T.

(b) Show that the replacement $\mu \to -\mu$ sends $\langle N \rangle \to M - \langle N \rangle$.

(c) For the case that $\langle N \rangle = M/2$ find μ at $T = 0$.

(d) By using your answer to part (c), verify that μ is in fact independent of T when $\langle N \rangle = M/2$. By using your answer to part (b) explain briefly the physical origin of this result.

(e) For the case that $\langle N \rangle = M/2$ find an expression for the heat capacity as a function of T. Sketch a graph of the heat capacity as a function of T, indicating significant features.

[Note: You may use without proof the following identities: $\cosh^2 x - \sinh^2 x = 1$ and $1 + \cosh 2x = 2\cosh^2 x$.]