CM In galaxies like our own, the visible stars and gas appear to be embedded in a much more massive cloud of invisible "dark matter" that interacts little if at all with ordinary matter and electromagnetic radiation. If the dark matter consists of χ-particles of mass M, and if these particles possess non-gravitational interactions of some kind, there are several ways that we might detect them:

a) **Accelerator production**: We may try to produce dark matter particles in an electron-positron collider, in which electrons and positrons of unequal lab frame energies E_- and E_+ collide head-on. If an electron-positron collision produces a pair of χ's via $e^+ + e^- \rightarrow \chi + \chi$, and assuming that the energies involved are large enough that we can neglect the masses of the electrons and positrons, what is the *largest* mass M of the χ particle that this collider can produce?

b) **Direct detection**: Dark matter χ-particles traveling at typical galactic orbital speeds should be constantly incident on nuclei in a terrestrial detector. If such a χ scatters elastically, the recoiling nucleus may be detectable. Suppose that a χ particle with lab-frame speed $10^{-3}c$ strikes a stationary Ge nucleus ($m = 69.9\,\mathrm{GeV}/c^2$). If our detector can only identify a nuclear recoil with kinetic energy greater than 10 keV in the lab frame, what is the *smallest* value of M (in units of GeV/c^2) that can be detected?

c) **Background**: A possible source of false-positive background events in the recoil experiment of part (b) is gamma radiation arising from radioactive decay of surrounding materials. What is the minimum energy that a photon must have if it can impart 10 keV of recoil kinetic energy to a Ge nucleus by scattering (not absorption)? Given that the energy of gamma rays from long-lived radioisotopes never exceeds 4 MeV, will this background be a problem?