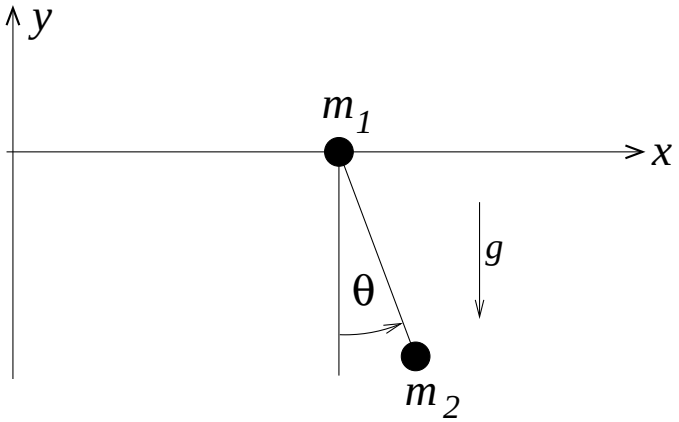
CM Consider a point-mass m_1 located at $(x_1, 0)$ that can slide on a frictionless horizontal rail oriented along the x axis. A pendulum of mass m_2, located at (x_2, y_2), is attached to m_1 with a massless rigid wire of length l. The motion of the pendulum is confined to the $x - y$ plane (see figure).



a) Take the position x_1 of m_1 and the angle θ that the pendulum makes with the vertical as the two generalized coordinates. Write down the Lagrangian L of this system in terms of the generalized coordinates and their time derivatives.

b) From the Lagrangian L, obtain the two equations of motion for the system.

c) Write down the momenta p_{x_1}, p_θ that are canonically conjugate to your generalized coordinates and use them to obtain the Hamiltonian of the system. Do you expect its value E to be a conserved quantity? Explain why or why not.

d) Write the expression, in terms of the generalized coordinates and their time derivatives, for the total horizontal momentum $P_x = m_1\dot{x}_1 + m_2\dot{x}_2$ of the system. Is P_x a conserved quantity? Why or why not?

e) Similarly write down the vertical component of the momentum $P_y = m_2\dot{y}_2$ in terms of the generalized coordinates. Is it a conserved quantity?