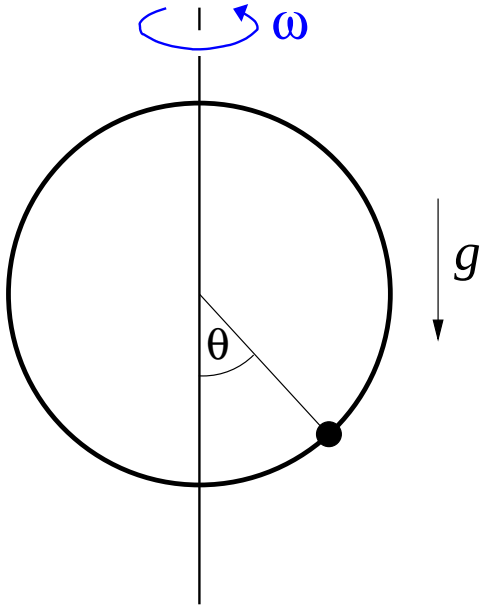
BCM. A bead of mass m slides without friction on a circular hoop of radius R. The hoop rotates about a vertical diameter with a constant angular velocity ω. The angle θ is measured from the lowest point on the hoop, and gravity acts downwards as shown in the figure.



a) For ω greater than some critical value ω_c the lowest point on the hoop is no longer a stable equilibrium point. Instead the bead can undergo small vibrations about a new equilibrium point $\theta_0 \neq 0$. Find both ω_c and, for $\omega > \omega_c$, the point $\theta_0(\omega)$.

b) Write down the exact equation of motion for arbitrary $\omega > 0$. For $\omega > \omega_c$ find the frequency $\Omega(\omega)$ of oscillations about the new stable equilibrium point.

c) Consider now the special case $\omega = \omega_c$. The bead is placed at $\theta = \pi/2$ and then released. What is the speed of the bead when it reaches the bottom of the hoop?