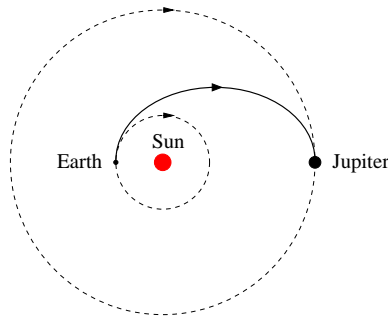


CMB. A spacecraft of mass m is to be sent from the Earth to Jupiter. It is decided to use a Hohmann-ellipse transfer orbit which is tangential to both the orbit of the Earth (assumed circular) at $r = R_1$, and to the orbit (also assumed circular) of Jupiter at $r = R_2$. [There is more than one way of solving this problem. If you decide to use any properties of Kepler orbits, you must derive them to gain full credit.]



- Find the orbital velocities v_1 and v_2 of the Earth and Jupiter. Your answer should be expressed in terms of R_1 or R_2 , Newton's constant G and the mass of the sun M .
- Find the departure orbital velocity v_D (measured with respect to the Sun and tangential to the Earth's orbit) that must be given to the spacecraft if it is to arrive tangential to Jupiter's orbit, and also the tangential velocity v_A at which it will arrive at this orbit. Again your answer should be expressed in terms of R_1 , R_2 , G and M .
- Find an explicit formula for the time that the journey from the Earth to Jupiter will take.

Hint: You may find one of the integrals

$$\int_{\alpha}^{\beta} \frac{dx}{x^2 \sqrt{(\beta-x)(x-\alpha)}} = \frac{\pi}{2} \frac{\alpha + \beta}{(\alpha\beta)^{3/2}}, \quad \int_0^{\pi} \frac{d\theta}{(\alpha + \beta \cos \theta)^2} = \frac{\pi\alpha}{(\alpha^2 - \beta^2)^{3/2}}$$

to be of use, or that the area of an ellipse with semi-major/minor axes a, b is πab .