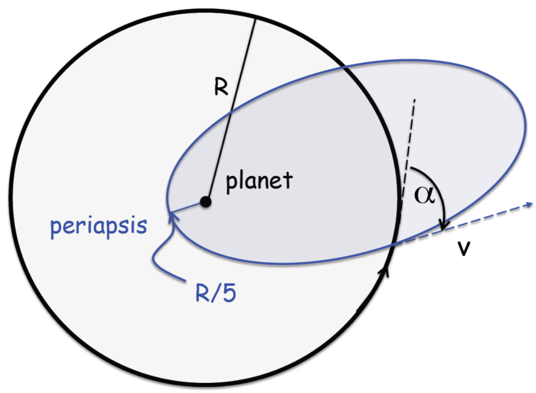
Q2 Consider the motion of a satellite of mass m in orbit about a much more massive planet.



a) Use polar co-ordinates r, θ. Without using a Lagrangian, derive expressions for the radial and angular components of the acceleration (a_r and a_θ respectively) of the satellite in terms of r, $\dot{r}$ and $\dot{\theta}$.

b) Use your expressions from part (a) to write down the equations of motion for the satellite. From these equations derive the laws of conservation of angular momentum and energy.

Assume now that the satellite moves in a circular orbit of radius R and at speed v. At some point in the circular orbit, an impulse of unknown origin causes the velocity vector of the satellite to be abruptly rotated through an angle α without changing its magnitude. As shown in the figure, this makes the satellite enter an elliptical orbit with periapsis (the distance of closest approach) of $R/5$. The plane of the elliptic orbit is the same as that of the circular orbit.

c) Find $v_{\text{periapsis}}$, the velocity of the satellite at closest approach to the planet.

d) Compute the angle α through which the satellite's velocity vector was turned by the impulse.