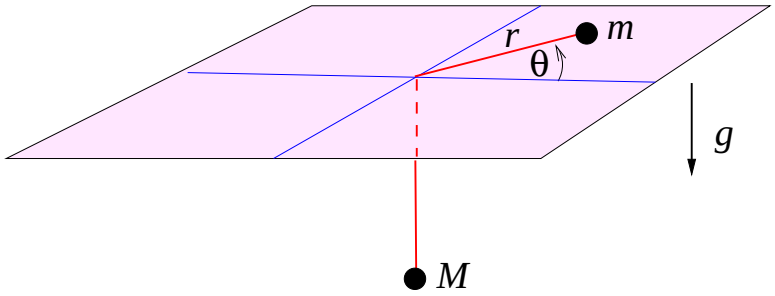
Q1 A particle of mass m slides on the surface of a smooth table. It is connected to a light inextensible string of length l that passes through a frictionless hole in the center of the table and is then attached to a second particle of mass M. You may assume that the mass M moves only in a vertical direction, and that the neither particle ever gets close to the hole in the table. Gravity acts vertically downward.



a) Using the co-ordinates r and θ, write down the Lagrangian that describes the motion of the two-particle system, and from it obtain the equations of motion for r and θ.

b) By identifying a suitable conserved quantity, reduce your two equations of motion for r and θ to a single second-order equation for r.

c) Reduce your second-order equation from part (b) to a first-order equation. This equation will contain two parameters that depend on the motion at time $t = 0$. Express the two parameters in terms of $r(0)$, $\dot{r}(0)$ and $\dot{\theta}(0)$.

d) Obtain the Hamiltonian of the system, and write down Hamilton's equations for the evolution of the co-ordinates and the generalized momenta.