CM A mass-spring system is suspended from a hook (see Fig. 1a). The mass m and the hook can only move vertically. The time dependent vertical position $y(t)$ of the hook is given. The spring is massless and obeys Hooke's law $F = -kx$, where F is the force, k is the spring constant, and x is the change in length of the spring from its equilibrium length.

Fig. 1a

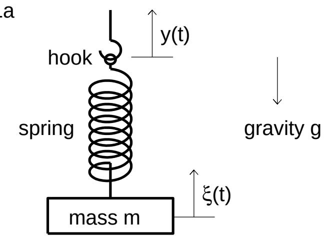
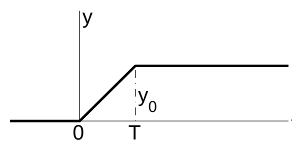


Fig. 1b



a) What is the Lagrangian for the system? Derive the equation of motion from the Lagrange function, or by any other method.

b) For $t < 0$ the hook and the mass are at rest, and the mass m is its equilibrium position. At $t = 0$ the hook starts moving upwards at a constant speed. At time $t = T$ it stops, *i.e.* $y = 0$ if $t \leq 0$, $y = y_0 t/T$ if $0 \leq t \leq T$ and $y = y_0$ if $t \geq T$ (see Fig. 1b). Show that for $t > T$ the displacement $\xi(t)$ of the mass from its initial position is

$$\xi(t) = a(1 - \cos\omega T)\cos\omega t + b\sin\omega T\sin\omega t + y_0.$$

Find the constants a and b.

c) Write the amplitude of the oscillation A as a function of y_0, ω, and T for $t > T$. Describe how the amplitude A depends on T for $\omega T \gg 1$.

d) Sketch the motion of mass m as a function of t. Label on your drawing T, y_0, and A.

e) Describe qualitatively how the evolution as a function of t would change, if the mass is in a viscous medium and the medium creates a small velocity-dependent friction force on the mass.

(Hint: $\sqrt{2 - 2\cos x} = 2|\sin x/2|$)