CM Consider a particle of mass m moving in a plane under the influence of a velocity-dependent potential. The system has Lagrangian

$$\mathcal{L}(\mathbf{r}, \mathbf{v}) = \frac{1}{2} m |\mathbf{v}|^2 - V_0 \frac{1 - \alpha \dot{r}^2}{r}$$

where $r = |\mathbf{r}|$ is the distance from the origin, $\mathbf{v} = \frac{d}{dt}\mathbf{r}$ is the velocity, V_0 and α are positive constants, and the dot denotes a time derivative.

a) Write the Lagrangian in polar coordinates (r, θ). From it find the corresponding canonical momenta p_r, p_θ, and also the Hamiltonian.

b) Show from the Hamiltonian equations of motion that the angular momentum p_θ and energy E are conserved.

c) Reduce the equations of motion to a single first order differential equation. (It is not necessary to solve this equation further.)

d) For a solution with a given positive energy E and angular momentum p_θ, what is the distance of closest approach to the origin? Why does your answer not depend on the value of α?