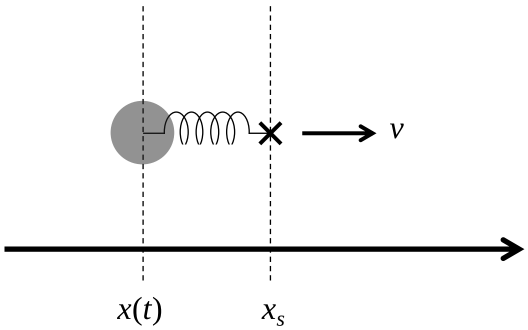
# 1

Consider a mass $M$ attached to an ideal, massless spring with stiffness $K$. The mass sits in a viscous medium with linear drag coefficient $\gamma$. Assume for simplicity that $M$ is confined to move along one dimension; its position is denoted by $x(t)$.




The end of the spring (marked by the **X**) is denoted by $x_s$. Assume for now that $x_s$ is fixed at the origin.

(a) Write down the equation of motion for the mass and find a general solution for $x(t)$.

(b) Suppose that the mass is displaced from its equilibrium and that its motion is highly overdamped; i.e. $\gamma/M \gg \sqrt{K/M}$. Calculate the time constants $\tau_1$ and $\tau_2$ (where $\tau_1 > \tau_2$) which characterize the decay of $x(t)$. Carry out your calculation to leading, non-vanishing order, i.e. neglecting terms of order $\gamma^3/MK^2$ and higher.

Now suppose that for $t \geq 0$, the end of the spring $x_s$ is moved to the right at a constant speed $v$, dragging the mass along, as shown in the figure. Suppose that the motion is highly overdamped as in part (b).

(c) Consider the limit as time $t \gg \tau_1$. Describe the motion of the mass relative to the end of the spring. Determine the power dissipated, i.e. the rate of change of the total energy, in this limit.

(d) Calculate an expression for $x(t)$ for $t \geq 0$, assuming the mass is initially at rest at position $x = 0$. Express your answer in terms of the time constants $\tau_1$ and $\tau_2$ in part b. Check that in the limit that $t \gg \tau_1$, you recover your answer to part (c).