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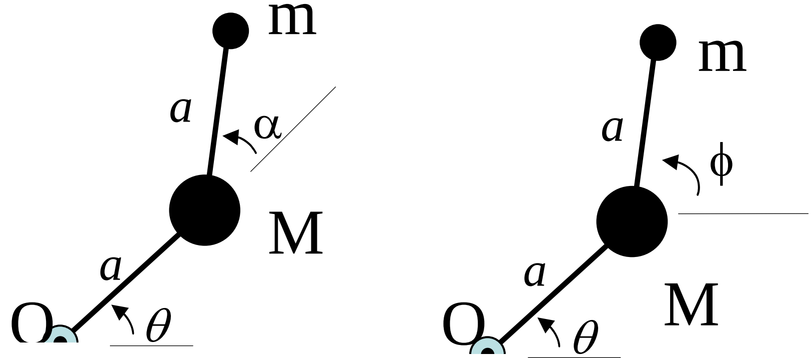
Two masses M and m are connected by two massless rigid rods of length a that are free to pivot in the (horizontal) frictionless plane of the paper. The point O is fixed. Gravity is not relevant.



(a) In terms of the two dynamical coordinates θ(t) and α(t) defined in the left hand figure, show that the Lagrangian is

$$\boldsymbol{L} = \frac{\boldsymbol{M a}^2}{2}\dot{\boldsymbol{\theta}}^2 + \frac{\boldsymbol{m a}^2}{2}\left[\dot{\boldsymbol{\theta}}^2\left(2+2\cos\boldsymbol{\alpha}\right)+\dot{\boldsymbol{\alpha}}^2+\left(2+2\cos\boldsymbol{\alpha}\right)\dot{\boldsymbol{\theta}}\dot{\boldsymbol{\alpha}}\right]$$

[*Hint: You may find it easiest to first construct L in terms of θ and ϕ defined in the right hand figure, and then change coordinates:* $\phi = \theta + \alpha$]

(b) Give physical or mathematical reasoning to identify all constants of the motion. Give expressions for them in terms of the dynamical coordinates θ and α and their time derivatives.

(c) Give expressions for the two generalized momenta p_θ and p_α in terms of the dynamical coordinates θ and α and their time derivatives.

(d) Derive the two coupled ordinary differential equations of the motion for θ(t) and α(t).

(e) Show that the steady solution $\theta = \Omega t$, $\alpha = 0$ with constant Ω satisfies these differential equations.

(f) Find the linearized equations that govern small perturbations ε,η, away from the steady solution, where

$\theta = \Omega t + \varepsilon(t)$; $\alpha = 0 + \eta(t)$

(g) What are the two characteristic frequencies for such perturbations?

(h) Below you see three snapshots of the system perturbed from the steady solution. (The steady solution $\theta(t) = \Omega t$, $\alpha(t) = 0$ is given by the dashed line.) Two of these correspond to the system being in a normal mode; one of them does not. Which of the panels (a), (b) or (c) correspond to normal modes? For those you identify as such, identify the frequency also.