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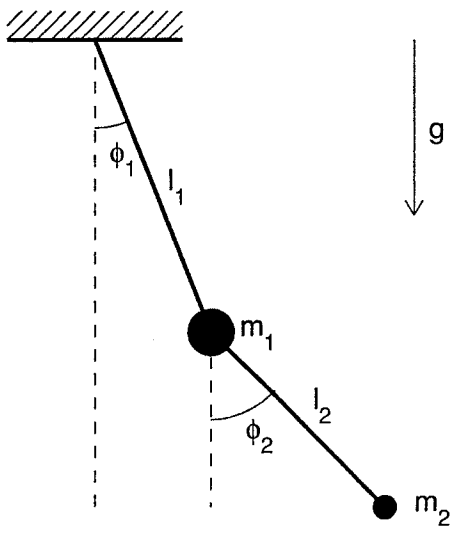

A double pendulum consists of one pendulum attached to another. Consider a double bob pendulum with masses $m_1 = M$ and $m_2 = m$ attached by rigid massless wires of length $l_1 = l_2 = l$. Further, let the angles the two wires make with the vertical be denoted ϕ_1 and ϕ_2, as illustrated above. Finally, let the acceleration due to gravity be g. The motion is in the x-y plane and frictionless.

1. Show that the Lagrange function is

$$L = \frac{1}{2}(M+m)\,l^2\dot{\phi}_1^2 + \frac{1}{2}ml^2\dot{\phi}_2^2 + ml^2\dot{\phi}_1\dot{\phi}_2 cos\,(\phi_1 - \phi_2) + (M+m)\,glcos\phi_1 + mglcos\phi_2$$

2. Determine the generalized momenta p_1 and p_2 that are conjugate to ϕ_1 and ϕ_2.
3. Determine the Euler-Lagrange equations of motion for ϕ_1 and ϕ_2.

Next we study small oscillations, i.e. $|\phi_1| \ll 1$ and $|\phi_2| \ll 1$.

4. Determine the characteristic frequencies. Sketch ϕ_1 and ϕ_2 as a function of time on the same graph to show the phase relation.

5. What are the limiting values for the frequencies for large M? What is special about this case?