Part A

A relativistic beam of monochromatic muons produced in the upper atmosphere is incident vertically on the earth's surface at velocity $v \approx c$, where c is the speed of light.

(1) Find the ratio of the muon number reaching the ground to the number at a height H above sea level. Assume that the beam moves at a constant velocity and that the attenuation of the beam is due entirely to the spontaneous decay of the muons. Express your answer in terms of the energy E of a muon as measured by an observer at rest on the ground, the muon rest-mass m_μ, and the muon rest-frame lifetime (i.e. decay time) τ_0.

(2) Explain in one sentence why the ratio must be the same for an observer comoving with the beam. Re-express your answer to (1) entirely in terms of parameters measured by a comoving observer.

Part B

The excitation energy of a free, excited nucleus at rest in the lab is $\delta\mathcal{E}$ and the mass of the excited nucleus is m. The nucleus de-excites by emitting a photon.

(1) Determine the frequency ν of the photon as measured in the lab, accounting for nuclear recoil. Express your answer in the form

$$\nu = a(\delta\mathcal{E}) + b(\delta\mathcal{E})^2 \ , \tag{1}$$

and find a and b.

(2) Explain in one sentence the *sign* of b.

(3) How does your answer change if the nucleus is rigidly attached to a crystalline lattice? Specifically, find a and b in this case.