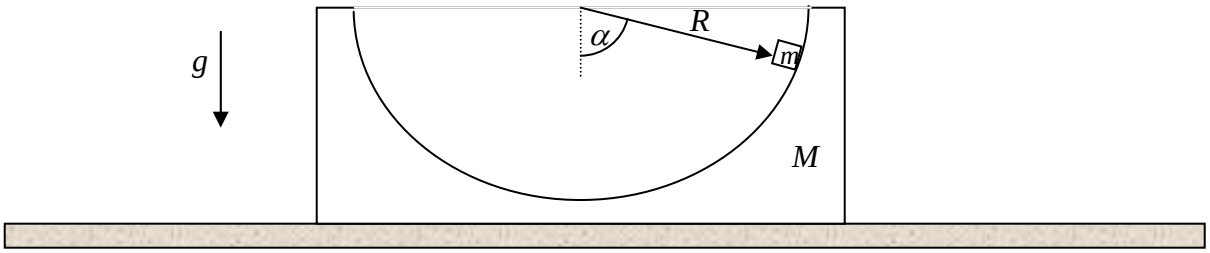
A particle of mass m is constrained to slide without friction on the surface of a smooth circular bowl of mass M with inner radius R as shown in the figure. The bottom of the bowl lies on a horizontal table and is free to slide without friction along the table. All motion is constrained to the plane of the page. Assume uniform gravitational acceleration.



(a) State the Lagrangian for this system.

(b) Derive the differential equations of motion for the particle and the bowl. You do not have to solve these equations.

(c) Determine the conserved quantities in this system and write equations describing these quantities.

(d) Determine the angular frequency for small oscillations about the equilibrium position. Show that in the limit of $M \gg m$, the result is the same as that of a simple pendulum of length R.