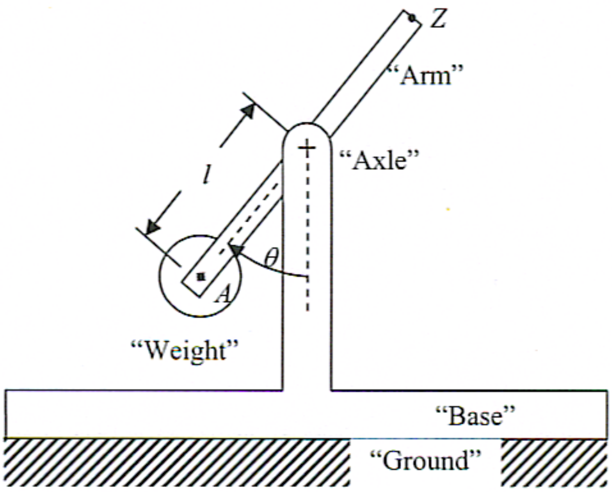
Consider the following simplified model of a catapult. As the weight falls, a projectile located near the point Z would be released. We will consider only the speed of point Z in this problem and neglect the actual projectile. In particular, we would like to analyze two variations of this design: what happens if the weight is allowed to rotate (rather than being fixed to the arm) and what happens if wheels are added to the base? We will consider only the part of the motion corresponding to small angles, θ, and all motion should be considered to be frictionless.



The arm is a rod of mass m, length $2l$ and negligible width. The weight is a disk of mass M, and radius R. The base has mass M_B.

(a) Calculate the frequency of small oscillations, ω, considering the base to be fixed relative to the ground and the weight to be fixed to the arm.

(b) Now calculate the frequency of small oscillations, ω_1, corresponding to the case where the base is still fixed relative to the ground, but the weight is free to rotate about axis A.

(c) Next, calculate the frequency of small oscillations, ω_2, corresponding to the case where the base is free to move (for simplicity you may imagine that the catapult is placed on a frictionless table) and the weight is still free to rotate about axis A.

(d) For each of the cases a), b) and c) and for $\theta(t = 0) > 0$, calculate the linear speed of point Z on the arm (relative to the ground beneath the base) for $\theta = 0$. Given these results, which modification(s) would a designer choose to maximize the range of the projectile and why?