



A cylindrical rod has length L and a circular cross-section of radius R in its own rest frame. It is oriented along the $+\hat{x}$ direction and is moving with speed $v_1 = \beta_1 c$ in the lab frame. A second rod which is identical to the first one (when at rest) is also oriented along the \hat{x} direction, and travels along the $-\hat{x}$ direction with speed $v = \beta_2 c$ in the lab frame. The figure above sketches the general situation, but does not necessarily indicate the correct dimensions of the rods. Assume β_1 and β_2 are both positive. The rods are made out of a solid, uniform material which has density ρ when at rest. The rods do not rotate.

- Calculate the length and the volume of the first rod in the lab frame.
- Calculate the energy and momentum of the first rod in the lab frame.
- The first rod has a light-emitting diode which emits light of frequency f_0 in all directions in its own rest frame. Find the frequency of this light as seen by an observer in the lab frame when the rod is heading straight at the observer. Find the frequency the observer sees after the rod has passed by.
- The rods collide head-on and stick together into a single lump. No radiation or additional particles are emitted from the lump after the collision. Calculate the lump's invariant mass. Calculate its momentum and speed in the lab frame.