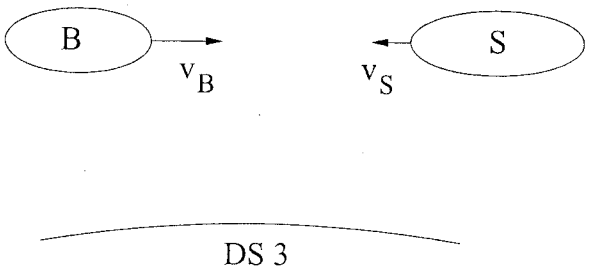
Astronauts Sally and Bob decide to settle one of their disagreements in a "paint duel". They fly their space shuttles on a collision course toward each other while keeping their shuttles at constant speed. Each shuttle is equipped with paint guns which are capable of emitting spray paint at high speed. Whoever gets the most paint on their shuttle loses. A worker from space station DS 3 is there to observe and referee the duel.

Note: Express times in units of s (seconds), velocities in units of c (speed of light), and distances in units of *light seconds*$= cs$.



In the frame of the space station the following events take place:

event 1: Sally fires her paint gun at $x = 9\,cs$ and $t = -2\,s$.
event 2: Bob fires his paint gun at $x = 0\,cs$ and $t = 0\,s$.
event 3: Bob's ship is hit by paint at $x = 4\,cs$ and $t = 5\,s$.
event 4: Sally's ship is hit by paint at $x = 5\,cs$ and $t = 6\,s$.

(a) Show that Sally's velocity in the rest frame of the space station is $v_S = -1/2\,c$ while Bob's velocity is $v_B = +4/5\,c$.

(b) Find the intervals $\Delta x'$ and $\Delta t'$ between events 1 and 2 in Sally's rest frame.

(c) Determine if it is possible for events 1 and 2 to be causally connected.

(d) Determine the velocity of the paint from Bob's paint gun in Bob's rest frame.

(e) Bob is trying to evade the rules of the paint duel, and sends a message on his communicator to a friend's shuttle hidden behind Sally's shuttle. Bob's communicator is using a frequency of $f = 10^{10}$Hz. Sally's receiver is able to receive communications in the frequency range of 3×10^9 to 3×10^{10}Hz. Determine whether Sally is able to receive Bob's message.