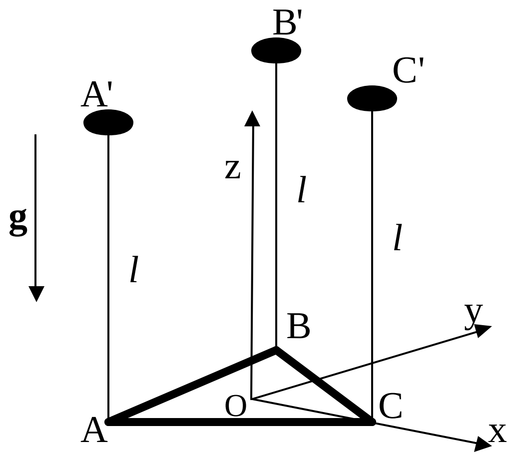
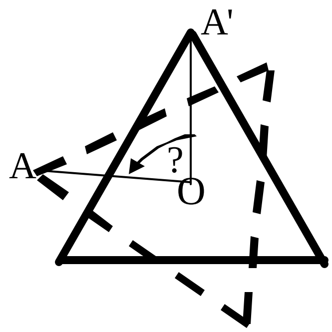


Top view



Consider an equilateral triangle ABC made of three metallic rods, each of mass m and length a. The triangle is suspended in a horizontal plane by three inextensible strings of negligible mass, each of length l. The strings are attached at each vertex of the triangle (points A, B, and C) and are connected to fixed points (A', B', and C') lying in a plane above the triangle. The plane A'B'C' is always parallel to the plane of the triangle. At equilibrium, each string is vertical. Consider small rotational oscillations about the z-axis (shown on the diagram), which goes through the center of the triangle (marked "O"). The rotation angle q and the direction of gravity (vector **g**) are shown in the figure.

(a) Find the moment of inertia of the equilateral triangle about the vertical z-axis passing through the center of the triangle. You may use without proof the result that the moment of inertia of a rod about the vertical axis passing through the center of mass of the rod and perpendicular to the rod is $ma^2/12$.

(b) Calculate the vertical displacement z of the triangle in the case when the rotation angle is changed from zero to q.

(c) For small angle rotations, show that the Lagrangian for this system can be written in the form $L = C_1 + C_2 q^2 + C_3 \dot{q}^2$, and find the constants C_2, C_3.

(d) Using Lagrange's equation (or otherwise) find the frequency of small oscillations w_{tr} of the suspended triangle in terms of the constants C_2 and C_3.