Two objects (mass $2m$ and m) are free to move in one dimension along a frictionless horizontal surface. They are connected to each other and to a fixed wall by two springs having stiffness $2k$ and k as shown in the diagram below. Choose the dynamical co-ordinates of the system to be the amount by which each spring is stretched beyond its equilibrium length. Consider only oscillations whose amplitudes are much less that the equilibrium lengths of the springs.
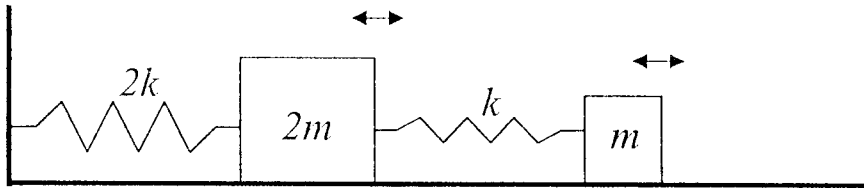
a) Show that the frequencies of the normal modes of the system are $\sqrt{2}\,\omega_0$ and $\omega_0/\sqrt{2}$, where $\omega_0^2 = k/m$.

b) Find the eigenvectors and sketch the normal modes of the system.

c) Now suppose you grab mass m and pull it to the right such that the rightmost spring is stretched an amount X_0 beyond its equilibrium length (and hence the leftmost spring is stretched a distance $X_0/2$ from its equilibrium length). You release the system from rest at $t=0$. Write an expression for the displacement of mass m as a function of time from its original "springs un-stretched" equilibrium position.