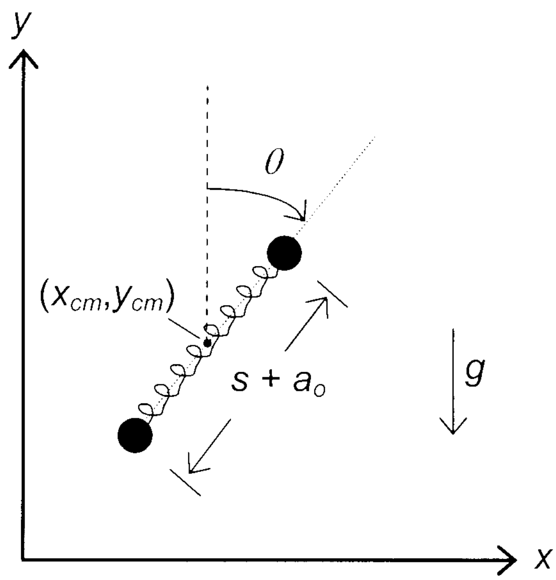
Consider a *classical* model of a diatomic molecule that is constrained to move in the vertical xy-plane, as shown to the right. The molecule consists of two particles of identical mass m, which are bound together with an effective spring constant k at a natural length a_o. The gravitational force is acting in the -y direction. The goal of this problem is to describe certain aspects of the motion of this molecule using the following generalized coordinates: the position of the molecule's center of mass (x_{cm}, y_{cm}), the angle θ the molecule makes with the y-axis, and the extension s of the molecule beyond its natural length a_o.



(a) Assuming that the molecule above is moving so that all of the generalized coordinates are changing with respect to time, **qualitatively sketch** on separate graphs the time-dependence of each of the generalized coordinates: x_{cm}, y_{cm}, s, and θ. Also, write down the quantities you expect to be conserved in this system, along with a brief explanation as to why these quantities are conserved.

(b) Write down the full Lagrangian for this system.

(c) Using the Lagrangian in part (b), identify the conserved quantities for this system, and write expressions for these conserved quantities in terms of the generalized coordinates and generalized velocities.

(d) Using the Lagrangian **or** another appropriate method, show that there are solutions for which the angular velocity $d\theta/dt$ is constant. For these solutions, write the molecule's extension s as a function of the magnitude of the molecule's angular momentum L.

(e) Using the Lagrangian **or** another appropriate method, write down the equation of motion for s for the case in which the vibrational amplitude is small, then determine the frequency of small oscillations as a function of the magnitude of the molecule's angular momentum L.