A point mass m glides without friction on a cycloid, which is given by $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)(0 \le \theta \le 2\pi)$. The apparatus sits in a uniform vertical gravitational field and the motion of the point mass m is in the x-y plane.

(a) Express the Lagrangian in terms of θ and $\dot{\theta}$.

(b) Determine the equation of motion.

(c) Let $u = \cos(\theta/2)$. Without assuming a small displacement of the point mass form the bottom of the cycloid, show that the exact solution for u satisfies the equation

$$\frac{d^2u}{dt^2} + \frac{g}{4a}u = 0.$$

(d) Find the general solution for $u(t)$.