Two masses m_1 and m_2 move on a frictionless horizontal surface. The two masses experience a mutual *attractive* force which is proportional to the cube of the distance (r) between them and directed along the line of separation between the two masses. Hence, the magnitude of the force is:

$$|\mathbf{F}| = k\,r^3 \qquad (k > 0)$$

There are no external forces acting on the system and you can neglect the gravitational force between the two masses.

(a) Write the Lagrangian using suitable coordinates for the center of mass and the relative position. Identify all constants of motion for this problem.

(b) Now, write down the Hamiltonian equations of motion for this problem. Show that the problem can be reduced to one degree of freedom. Explicitly state the equation of motion for that degree of freedom. Find and sketch the corresponding effective potential. Note that you do not need to solve (or integrate) the equations of motion.

(c) If the relative motion of the system describes a circle of radius r_0, find the total energy and the angular momentum of the system. Express your answers in terms of the masses, k, and r_0.

(d) Find the frequency of small radial oscillations, if the system is slightly perturbed from circular motion.