CM FALL 01 B

A star is orbiting in a galaxy with gravitational potential $\psi(r)$ ($\psi(r)$ is not necessarily $\propto 1/r$); here $r$ is radius in the usual spherical coordinates $r$, $\theta$, $\phi$.

a) Show that the star's orbit is confined to a plane.

b) Suppose that the star is on a circular orbit about the origin with radius $r = r_0$. Write down the Lagrangian for the star and use it to find the star's orbital speed v.

c) Suppose that the star is initially on a circular orbit, but is subsequently given a small impulse in the radial direction. Under what circumstances is the orbit stable?

d) Will all orbits still be confined to a planes if the potential is perturbed via $\psi(r) \rightarrow \psi(r)\,(1 + \varepsilon \cos(\theta))$?