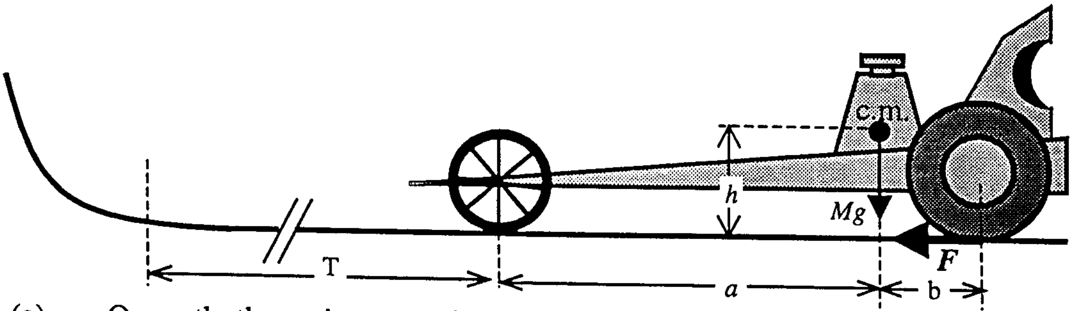
CMspring00B

A specially equipped racecar starts out at rest and accelerates continuously along a long track until it reaches a speed great enough to be launched into space. In space, a different, novel "engine" is used to propel the car to relativistic speeds.



(a) On earth, the car's conventional engine is capable of applying an arbitrary torque to the rear wheels. Under the condition of no slipping, this produces the force, ***F***, on the car as indicated in the figure. Determine the maximum possible acceleration of the car, a_{max}, in terms of the maximum coefficient of static friction, μ, and the gravitational acceleration, g. The front wheels of the car remain on the ground at all times.

(b) The weight distribution of the car is designed to achieve the maximum acceleration, a_{max}. With the center-of-mass (c.m.) of the car located a distance b in front of the rear wheels and a distance h above the road surface, find the relationship between b and h that produces the acceleration a_{max} found above.

(c) **Derive** the expression for the escape velocity of the car in terms of g and R_E, the radius of the earth. Use this to compute the length of the track, T, necessary to accelerate to the escape velocity starting from rest under the assumming that the car can maintain the constant acceleration a_{max}. (Assume that the car's mass does not change in time and that μ is large enough such that $T < R_E$.)

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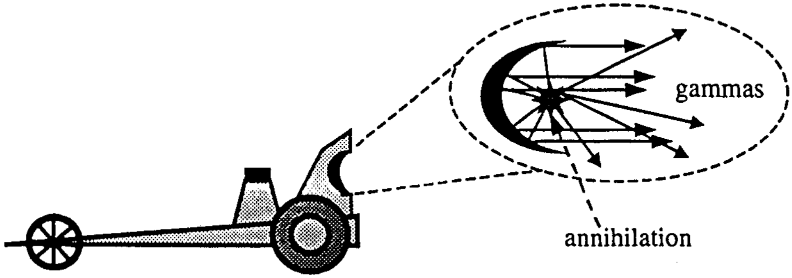

In space, a novel engine is now used to propel the car to the relativistic speed, v. The fuel for this engine is composed of equal parts matter and antimatter which are permitted to annihilate at the focal point of a parabolic mirror at the rear of the car. The annihilation turns the fuel mass, m_{fuel}, into gamma rays with 100% efficiency. All gamma rays emitted in the forward hemisphere (toward the front of the car) strike the mirror and are perfectly reflected out the back, antiparallel to the car's motion. Gamma rays initially emitted in the backward hemisphere escape the car without hitting the mirror (see figure for examples).

(d) What fraction α of the fuel mass is converted into the car's forward momentum?

(e) Find the final relativistic γ factor where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ of the car with v measured with respect to the Earth's inertial frame. Your answer should only contain α, m_{fuel} and m_{car}, the mass of the empty car.