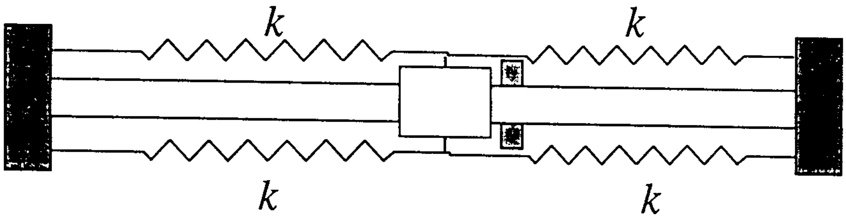
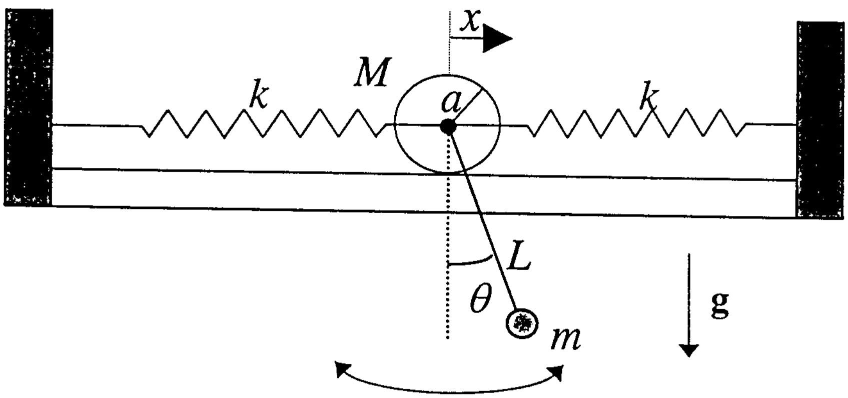
CMSpring00A

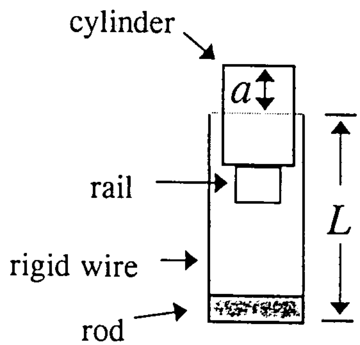
Top View



Side View



End View



A solid cylinder of radius a and mass M rolls along a horizontal rail without slipping. The center of the cylinder is attached to the walls by four massless springs, each having a spring constant k. A rod of negligible radius and mass m is hung from a frictionless and massless axle at the center of the cylinder by means of rigid, massless wires of length L. The rod is free to swing in response to gravity as shown above. The moment of inertia of the cylinder about the axle is $I = {}^1\!/_2 Ma^2$.

(a) Write down the kinetic energy T of this system in terms of the parameters given.

(b) Write down the potential energy V of this system in terms of the parameters given.

(c) Write down the coupled equations of motion describing the cylinder's displacement from equilibrium along the rail, X, and the hanging rod's angle from equilibrium, θ. Do not attempt to solve or simplify the equations at this point.

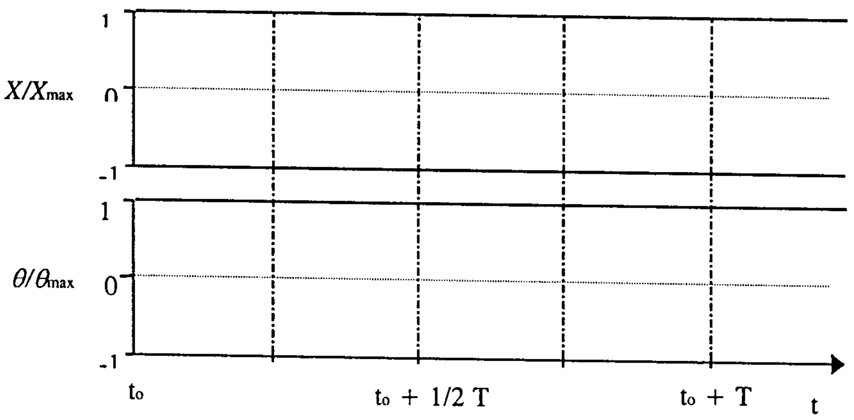
[continued on next page]

(d) After making <u>both</u> of the following two simplifying assumptions to your answer in part (c), determine the frequencies of the normal vibrational modes of this system:

(i) Let $M = 2m$ and $kL = mg$, where g is the acceleration due to gravity, and

(ii) assume both the pendulum and cylinder have small oscillations.

(e) Assume that $X(t=t_o) = X_{max}$, where X_{max} is the maximum positive displacement of the cylinder from equilibrium. For *each* of the normal vibrational modes, sketch in your answer booklet both the normalized cylinder displacement, X/X_{max}, and the normalized rod swing angle, θ/θ_{max}, as a function of time using the same time axis for both quantities. Sketch only for one full period T of the system's motion (see graph below).



(f) Identify which of the normal modes in part (e) has the lower frequency.