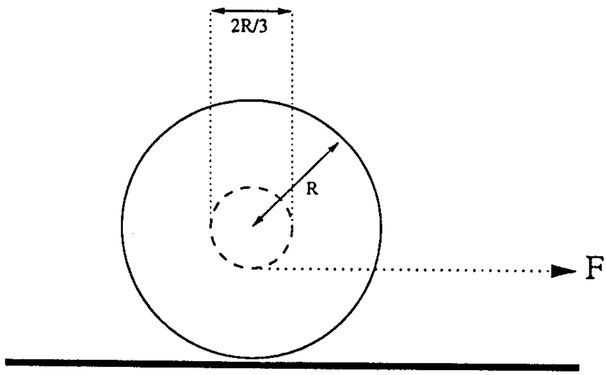
CM Fall99B

The three parts of this problem are independent.

(a) A mass m moves in a circular orbit of radius r_0 under the influence of a central force whose potential is $-km/r^n$ with $k > 0$. Show that the circular orbit is stable under small perturbations if $n < 2$.

(b) A yo-yo rests on a level surface. A gentle horizontal pull (see figure) is exerted on the string so that the yo-yo rolls without slipping.
(i) Which way does it roll?



(ii) The inner shaft of the yo-yo has one third the radius of the outer disk, which is R, and the moment of inertia of the yo-yo about its center is $I = \frac{1}{2}MR^2$ where M is the mass of the yo-yo. What is the torque about the center of the yo-yo? Express your answer in terms of F, M, R, the acceleration due to gravity g and the coefficient of friction μ.
(iii) Use the no-slip condition to obtain the (linear) acceleration in terms of F.
(iv) What is the minimum value of the coefficient of friction, $\mu_{\rm min}$, such that the yo-yo will roll without slipping?

(c) Consider a relativistic 1D harmonic oscillator: a particle of rest mass m moving in a potential $\frac{1}{2}m\omega^2x^2$. Use conservation of energy to solve for the velocity $v(x)$. Argue that the first order relativistic correction to the period of the oscillator is

$$\tau = \frac{2\pi}{\omega}\left[1 + (\text{const.})\frac{\omega^2 a^2}{c^2} + \cdots\right]$$

where a is the amplitude of the oscillator and c is the speed of light. Is the constant positive or negative, i.e. does the period get longer or shorter once relativistic effects are included?