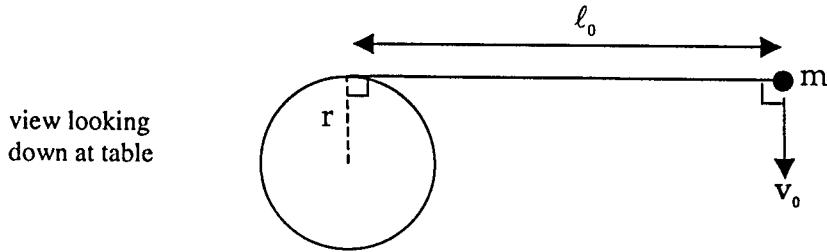A peg of radius r is firmly attached to a smooth horizontal table. A light inextensible string of length ℓ_0 connects a mass m to the peg and is initially tangent to the point of contact. The mass is given a horizontal velocity v_0 at time t = 0 in a direction perpendicular to the tangent line, as shown.

The string remains taut and proceeds to wind around the peg as the mass slides without friction on the table. After a short time some of the string has wound up and the system looks like:
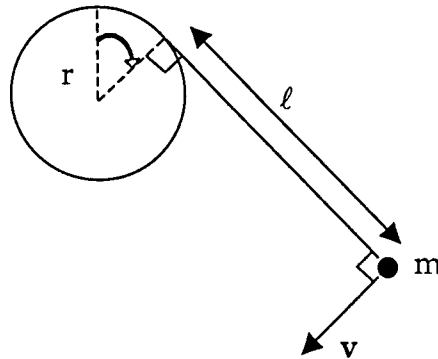
A) Is the kinetic energy of the mass conserved? Is the angular momentum of the mass conserved? Give your reasoning.

B) Write down the Lagrangian for the system in terms of ℓ and $d\ell/dt$.

C) By using the Lagrangian, or otherwise, determine the remaining length ℓ of the string as a function of time.