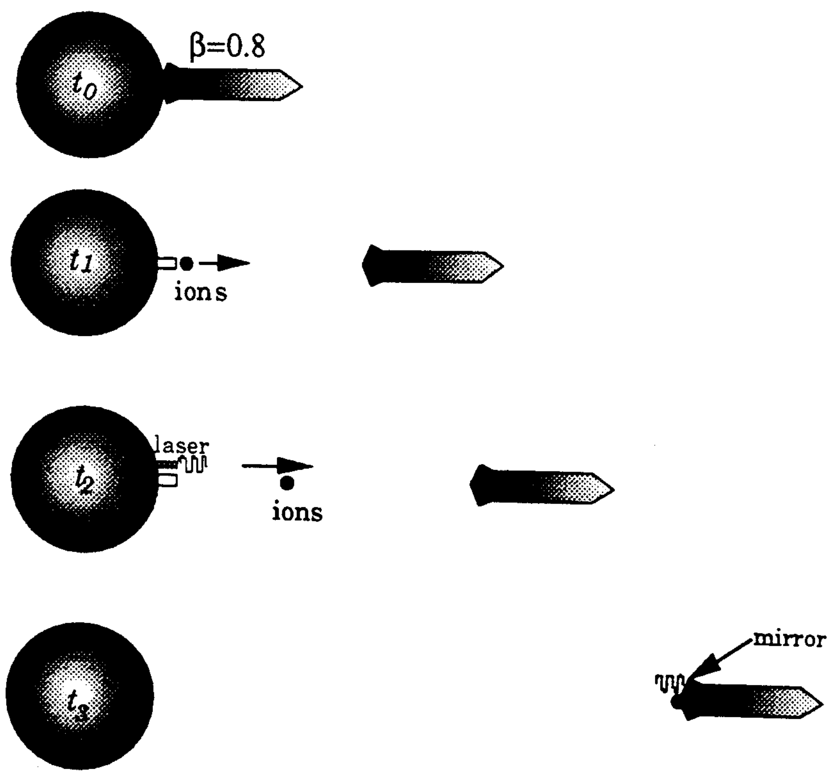
CM7a1196A

Notes: All unprimed quantities are measured *in the earth's reference frame.*

You may find the following example of the Lorentz transformation useful:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}, \text{ with } \beta = v/c \text{ and } \gamma = \frac{1}{\sqrt{1-\beta^2}}.$$

A rocket blasts off from the earth at time $t_0=0$. The rocket reaches a velocity corresponding to $\beta=0.8$ in a negligible time and continues at this speed throughout the problem. At t_1, a hostile enemy fires a packet of 10^{12} radioactive ions at the ship, each with rest mass m_0. At t_2, a pulse from a laser is sent from earth to warn the rocket of the impending doom. The warning is received at the rocket at time t_3, exactly at the moment that the ion beam also arrives.



(continued overleaf)

(a) The momentum of each ion in the packet is $p_{ion} = 3m_0c$. What is the total energy, E, of the radioactive ion burst?

(b) Let t_1 = 15 hours. Find values for t_2 (the time the laser is fired) and t_3 (the time the ion beam and the laser both arrive at the rocket).

(c) The radioactive ions have a proper lifetime, τ_0, of 10 hours. Assume that ions which decay miss the rocket and that those which strike the rocket are fully absorbed. *In the reference frame of the rocket* how much total energy, E', is absorbed when the ion packet strikes?

(d) A mirror placed at the back of the rocket reflects part of the laser light back to earth. The wavelength of the light emitted by the laser in its rest frame is 337 nm. *Derive* the relativistic Doppler shift expression starting with the basic Lorentz transformation. Next, apply your result to determine the wavelength of the reflected laser light as it would appear from earth.