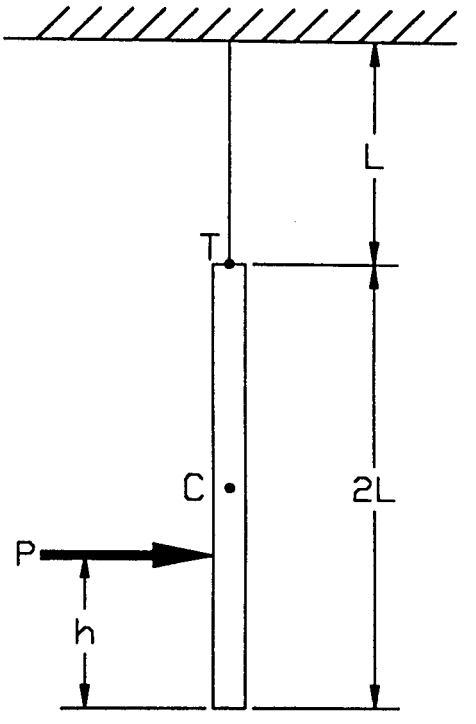
CMSpring96B

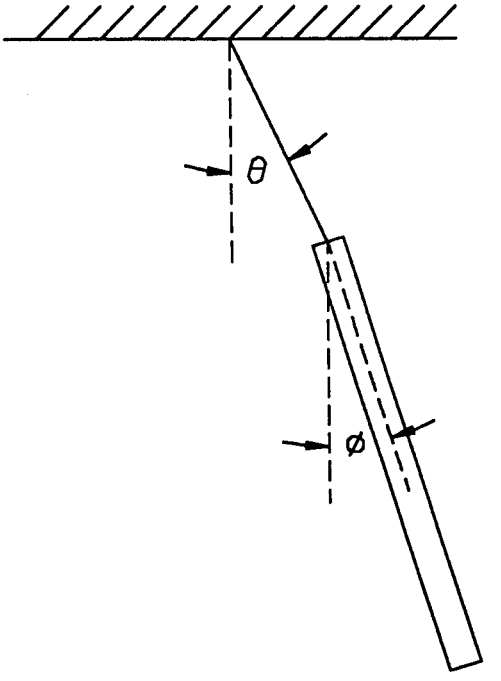
A uniform rod of mass M and length 2L is suspended at one end from the ceiling by a massless string of length L. An impulse of magnitude P is applied to the rod in a horizontal direction at a distance h above the bottom end of the rod.



(a) Calculate the speed of the center of mass C of the rod immediately after the impulse.

(b) Calculate the speed of the top end of the rod T immediately after the impulse. Give your answer in the form $v_T = AP/M$, where A is an expression which depends on h and L only. (Hint: the moment of inertia of the rod about its center of mass is $I = (ML^2)/3$.)

For the rest of the problem, assume that the impulse is very small. As a result, the string remains taut, and the angles θ and ϕ defined in the figure remain small at all times. Use the usual small angle approximation.



(c) Write down expressions for the kinetic energy T and potential energy V in terms of θ and ϕ and their time derivatives.

(d) Calculate the oscillation frequencies of the normal modes of the system.

(e) For a particular value of h ($0 \le h \le 2L$), the motion resulting from the impulse occurs at only one of the two frequencies determined in part (d). Calculate h/L (a numerical value), and identify the corresponding frequency.