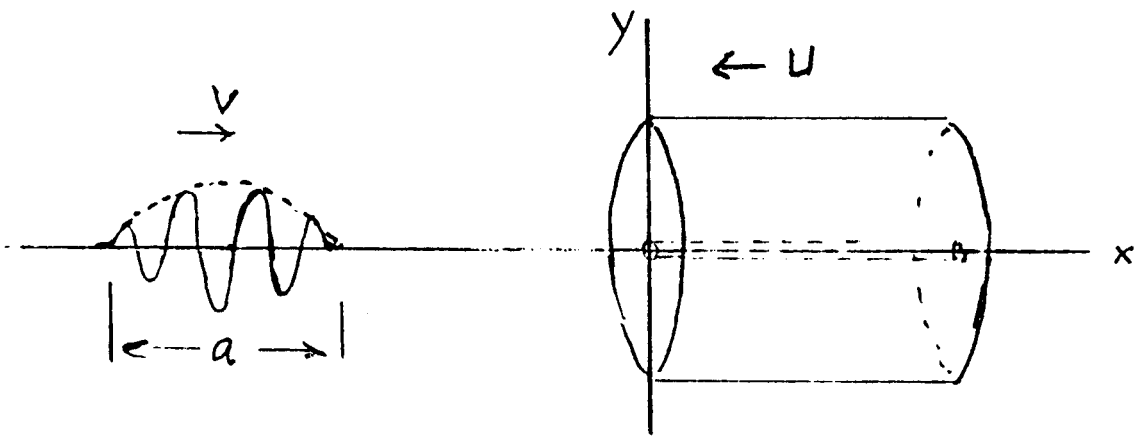
CM7all95B

A string with mass per unit length μ under tension T lies on the x-axis. It passes through a solid cylinder of mass M with a small opening along its axis. (See figure.) The cylinder is constrained to move freely along the x-axis. At $t = 0$, a pulse of short duration, $y = f(x)$, and height large compared with the diameter of the opening is generated on the left of the cylinder and moves towards the right with velocity $v = \sqrt{T/\mu}$. At the same instant, the left edge of the cylinder is located at $x = 0$ and is moving toward the left with velocity $U < v$. You may assume that M is large so that the velocity of the cylinder is not affected by the impact of the pulse, and that there is no transmission of the pulse across the left boundary of the cylinder.



(a) Given that the string has zero transverse displacement inside the cylinder, write down the appropriate boundary condition for the string at the left edge of the cylinder.

(b) Using the general solution of the wave equation and the boundary condition in (a), show that for $t > 0$ the wave has the form

$$y = \begin{cases} f(x-vt) - f(-A(x+vt)) & x < -Ut \\ 0 & x > -Ut \end{cases}$$

Determine the constant A.

(c) Show that the ratio of the energy of the pulse before its reflection from the cylinder to that after its reflection is 1/A.